

Rayleigh Waves Excited by the Discontinuous Advance of a Rupture Front

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Analytical results are presented for Rayleigh waves excited by a sudden change in the rate of growth of a subsurface zone of rupture. The curved rupture front advances across an inclined plane. The rupture can be brittle or cohesive tractions can act at its front. The analysis consists of two parts: First, ray theory is used to calculate wavefront approximations to the waves emitted when the rupture front speed suddenly changes. Secondly, a representation integral for the Rayleigh wave, where the integration is performed over a surface enclosing the rupture front, is constructed by using the emitted waves in combination with an appropriate Green's tensor. This integral is evaluated asymptotically. Synthetic accelerographs are constructed which illustrate how the rupture process, and the geometry of the rupture front and the fault plane affect the excitation of Rayleigh waves.

1. INTRODUCTION

Starting and stopping phases of a sliding event tend to manifest themselves primarily at the high-frequency end of the spectra of radiated seismic waves [Aki and Richards, 1980, chapter 15]. Madariaga [1977] and Achenbach and Harris [1978] have approximated the waves radiated from starting and stopping phases by using results of dynamic crack propagation in combination with elements of ray theory. Harris and Achenbach [1981] have further analyzed the reflection of these emitted waves from a free surface to estimate the ground motion in the near field. In the present paper we investigate Rayleigh waves excited on the free surface by stopping and starting phases of the faulting event.

At some distance from the epicenter the ground motion is primarily due to surface waves. Though it is possible to analyze surface waves using ray theory [Keller and Karal, 1964], this requires the tracing of rays in complex space. In this paper we follow a simpler method that Harris and Achenbach [1983] used earlier to calculate the Love waves excited by starting and stopping phases. In this method the surface wave is expressed by means of a representation integral over a surface $\mathcal{S}_i$ that encloses the rupture front. This representation integral is constructed by using the waves emitted by the changes in rupture-front speed, which were computed previously by ray methods, in combination with the Rayleigh-wave component of the Green's tensor for the half space. The representation integral is then evaluated by the method of stationary phase to give an explicit form for the Rayleigh wave.

An earlier paper that shares some of the same objectives as this one was written by Mal [1972]. Mal calculated the Rayleigh waves excited by a two-dimensional dislocation model of a moving thrust fault rather than by a crack-propagation model.

2. RADIATION FROM A RUPTURE FRONT

In earlier work Achenbach and Harris [1978] developed a crack-propagation model of an expanding slip zone that de-

scribes three basic features: the speed with which the rupture front advances can change; the curvature of the rupture front can be arbitrary; the rupture can be brittle or cohesive tractions can act at the rupture front. In this model the rupture front is idealized as a curve in the fault plane marking the transition from continuous displacements to conditions of sliding, and the change in rupture-front speed is idealized as discontinuous. The waves emitted by a change in the rupture-front speed were approximated near their wavefronts.

Let us consider a region of dip slip in a two-dimensional geometry. The longitudinal L and transverse T waves emitted from the rupture front when its speed suddenly slows from v_1 to v_2 at $t = 0$ are approximated near their wave fronts by

$$v_r(r, \theta, t) = (c_L/2\pi r)^{1/2} E_L^{TV}(\theta) e(t - r/c_L) \quad (1)$$

$$v_\theta(r, \theta, t) = (c_T/2\pi r)^{1/2} \{ \text{Re} [E_T^{TV}(\theta)] e(t - r/c_T) + \text{Im} [E_T^{TV}(\theta)] f(t, r/c_T) \} \quad (2)$$

where

$$E_\alpha^{TV}(\theta) = -F_\alpha(\theta) \sin 2\theta \quad \alpha = L \quad (3a)$$

$$E_\alpha^{TV}(\theta) = -F_\alpha(\theta) \cos 2\theta \quad \alpha = T \quad (3b)$$

$$F_\alpha(\theta) =$$

$$\frac{\left(\frac{c_T}{c_\alpha}\right)^3 \left[\frac{(c_R + v_1)}{c_T}\right] S_-(-s_{12}) \left[\frac{(v_1 - v_2)}{(v_1 + c_T)}\right]^{1/2} \left[1 + \left(\frac{c_T}{c_\alpha}\right) \cos \theta\right]^{1/2}}{\left[1 - \left(\frac{v_1}{c_\alpha}\right) \cos \theta\right] \left[1 - \left(\frac{v_2}{c_\alpha}\right) \cos \theta\right]^{1/2} \left[1 + \left(\frac{c_R}{c_\alpha}\right) \cos \theta\right] S_-(\zeta_\alpha^+)}$$

$$\alpha = L, T \quad (4)$$

$$e(t - r/c_\alpha) = H(t - r/c_\alpha) \frac{F_0 v}{2\pi^{1/2}} \int_{r/c_\alpha}^t \frac{(t-u)^{(v-2)/2}}{(u-r/c_\alpha)^{1/2}} du \quad (5)$$

$$\alpha = L, T$$

$$f(t, r/c_T) = H(t - t_h) \frac{F_0 v}{2\pi^{1/2}} \int_{t_h}^m \frac{(t-u)^{(v-2)/2}}{(r/c_T - u)^{1/2}} du \quad (6)$$

Here v_r and v_θ are the r and θ components of the particle velocity (the $[r, \theta]$ coordinate system is shown in the inset to Figure 1). They were previously calculated by Achenbach and Harris [1978, equation (14) and (27)]. Further, the terms E_L^{TV}

and E_T^{TV} , which we call emission coefficients, are equal to $(2\pi/c_L)^{1/2} D_L^{TV}$ and $(2\pi/c_T)^{1/2} D_T^{TV}$ where D_L^{TV} and D_T^{TV} are given by equations (15) and (28) of the earlier paper. Note, however, that equations (17), (29) and (34) of this earlier paper should be corrected. Equation (17) should be multiplied by $[1 - (v_2/c_L) \cos \theta]^{1/2}$, and (29) and (34) should both be multiplied by $[1 - (v_2/c_T) \cos \theta]^{1/2}$. Equations (17) and (29) affect D_L^{TV} and D_T^{TV} , respectively. Embedded in the expressions for the emission coefficients is the function $S_-(\zeta)$. This function is given in Appendix C. Its arguments ζ_a^+ and s_{12} are given by

$$\zeta_a^+ = (-\cos \theta/c_a)[1 - (v_2/c_a) \cos \theta]^{-1} \quad (7)$$

and $s_{12} = (v_1 - v_2)^{-1}$. The function $S_-(\zeta_T^+)$ is real for $|\theta| < \cos^{-1}(-c_T/c_L)$, where θ is assumed real, and it is complex otherwise. Thus in (2) it is necessary to express v_θ as a sum of real and imaginary parts. The functions $e(t - r/c_\alpha)$ ($\alpha = L, T$) and $f(t, r/c_T)$ are called the waveform functions; the first two were given by equation (26) of the earlier paper [Achenbach and Harris, 1978], while the last is new. The branch of the square root in (5) and (6) is taken so that $(r/c_T - t)^{1/2} = -i(t - r/c_T)^{1/2}$. In (6) the time $m = \min(t, r/c_T)$ and the time t_h is the headwave arrival time. The waveform functions depend upon the parameters F_0 and v . The parameter F_0 governs the magnitude of the slip near the rupture front. The parameter v governs the way in which the rupture occurs: $v = 1$ if the rupture is brittle, otherwise $v > 1$. The three constants c_L , c_T , and c_R are the L , T , and Rayleigh wave speeds.

The longitudinal and transverse waves excited by a sudden increase in the rupture front speed from v_2 to v_1 can be calculated from the above expressions by multiplying F_0 by -1 . It is to be understood that the particle velocities for either a decrease or increase in the rupture front speed are to be added to the particle velocities still existing because of previous rupture processes. Care must be taken when differentiating or integrating v_r and v_θ with respect to time because they represent a

wave field radiated from a source that moves with speed v_2 [Achenbach and Harris, 1978, equation (A1)]. To calculate the particle acceleration a_r from v_r , not only must $e(t - r/c_L)$ be differentiated but also the whole expression must be multiplied by $[1 - (v_2/c_L) \cos \theta]^{-1}$. A similar comment applies to calculating a_θ from v_θ .

To extend the two-dimensional results given by (1) and (2) to a three dimensional configuration, we consider a rupture front $\mathcal{C}$ of a planar slip zone advancing with a velocity v and assume that the slip is normal, at least locally, to the rupture front. The fault plane makes an angle ϕ with the normal to the surface of the half space. An instantaneous position of $\mathcal{C}$ is shown in Figure 1. In general the radius of curvature of $\mathcal{C}$, which is defined by ρ , may vary along the rupture front. A point on the rupture front $\mathcal{C}$ is defined by the polar angle ψ . The unit vector $\mathbf{t}(\psi)$ is tangential to $\mathcal{C}$. The plane $\mathcal{N}(\psi)$, which is normal to $\mathbf{t}(\psi)$, contains a polar coordinate system with its center on the intersection with $\mathcal{C}$. The plane $\mathcal{N}(0)$ and its coordinate system (r, θ) are shown in Figure 1. A second coordinate system (x, y, z) is located with its origin at the intersection of the fault plane with the surface of the half space; thus point B has coordinates $(-d \tan \theta, d, 0)$.

At a change in the rupture front speed from v_1 to v_2 , the wave front approximations to the waves from a point P on the rupture front can be represented by fans of rays in the plane $\mathcal{N}(\psi)$ through P normal to the rupture front. In two dimensions the particle velocities near their wavefronts are given by (1) and (2). In three dimensions these canonical solutions must be adjusted for the curvature of the rupture front by multiplication by a factor $(1 + r/\rho)^{-1/2}$, where

$$\bar{\rho}(\psi, \theta) = \rho(\psi)/\cos \theta \quad (8)$$

In subsequent work the Fourier transforms of (1) and (2) will be needed. The transform pair are given as follows:

$$f(\omega) = \int_0^\infty e^{i\omega t} f(t) dt \quad (9a)$$

$$f(t) = \text{Re} \left\{ \pi^{-1} \int_0^\infty e^{-i\omega t} f(\omega) d\omega \right\} \quad (9b)$$

where ω is the transform variable, which can be taken as complex to ensure convergence.

Equations (1) and (2) are wave front approximations; in the frequency domain they correspond to high-frequency or far-field approximations. Therefore when calculating the Fourier transforms of the waveform functions only the high-frequency approximations are needed. The resulting frequency domain expressions for the particle velocity components are

$$v_r(r, \theta, \omega) = (c_L/2\pi r)^{1/2} E_L^{TV}(\theta) e^{ik_L r} e(\omega) \quad (10)$$

$$v_\theta(r, \theta, \omega) = (c_T/2\pi r)^{1/2} E_T^{TV}(\theta) e^{ik_T r} e(\omega) \quad (11)$$

where

$$e(\omega) = F_0 \Gamma(\frac{1}{2}v + 1) (-i\omega)^{-(v+1)/2} \quad (12)$$

and $k_a = \omega/c_a$ ($\alpha = L, T$). Note that the Fourier transforms of the particle accelerations a_r and a_θ are given by $\{-i\omega/[1 - (v_2/c_L) \cos \theta]\}v_r$ and $\{-i\omega/[1 - (v_2/c_T) \cos \theta]\}v_\theta$, respectively.

3. EXCITATION OF RAYLEIGH WAVES

Rayleigh waves excited by emission from a propagating rupture front will be investigated, first in the frequency domain and

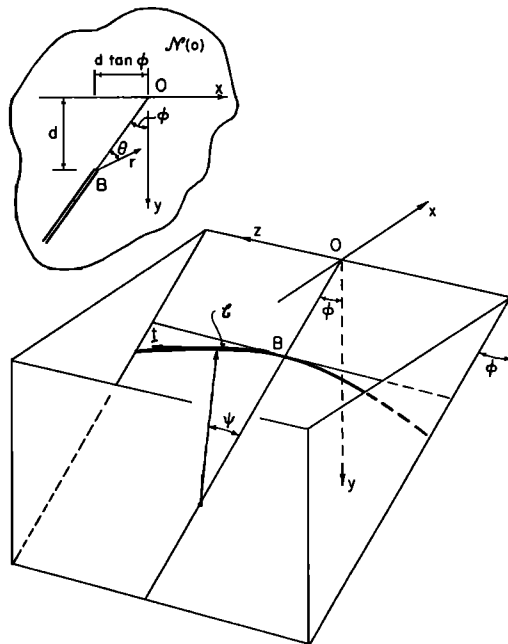


Fig. 1. Rupture front $\mathcal{C}$ of a planar slip zone advancing across an inclined fault plane with a velocity v normal to the front. At the instant shown, point B , which lies in the symmetry plane $\mathcal{N}(0)$, is a distance d below the surface and is moving toward the origin O .

then in the time domain. The overall geometry is shown in Figure 1. In outline the calculation, which is explained in more detail by Harris and Achenbach [1983], begins as follows. Using the Green's displacement and stress tensors, $u_{i;k}^G$ and $\tau_{ij;k}^G$, for the elastic half space [Achenbach *et al.*, 1982, chapter 3] and the reciprocity identity, the particle displacement u_k is represented by the surface integral

$$u_k(\mathbf{x}) = \int_{\mathcal{S}_i} [u_{i;k}^G(\mathbf{x}', \mathbf{x}) \tau_{ij}(\mathbf{x}') - u_i(\mathbf{x}') \tau_{ij;k}^G(\mathbf{x}', \mathbf{x})] n_j dS(\mathbf{x}') \quad (13a)$$

where $\mathcal{S}_i$ is a surface enclosing the rupture front ($\mathcal{S}_i$ need not be contained within the half space) and n_j is a unit normal pointing inward. The displacement u_k within the integral is approximated by u_k^I , the wave field emitted by the rupture in an unbounded medium. The Green's tensors within the integral are broken up into a sum of a Rayleigh wave term, indicated by $u_{i;k}^{GR}$ and $\tau_{ij;k}^{GR}$, and body wave terms. The integral containing the former gives the Rayleigh wave excited by the rupture. Thus the Rayleigh wave displacement u_k^R is given by

$$u_k^R(\mathbf{x}) = \int_{\mathcal{S}_i} [u_{i;k}^{GR}(\mathbf{x}', \mathbf{x}) \tau_{ij}^I(\mathbf{x}') - u_i^I(\mathbf{x}') \tau_{ij;k}^{GR}(\mathbf{x}', \mathbf{x})] n_j dS(\mathbf{x}') \quad (13b)$$

We are interested in calculating u_k^R in the far field; therefore, we need only the far-field approximations to $u_{i;k}^{GR}$ and $\tau_{ij;k}^{GR}$. The former is given by (A1) in Appendix A. Further, to evaluate (13b) we place the surface $\mathcal{S}_i$ in the far field also. There we may approximate u_i^I by the far-field approximations of the previous section. For the particular case in which the rupture front speed slows from v_1 to v_2

$$u_i^I(\mathbf{x}) = (-i\omega)^{-1} \{ [1 - (v_2/c_L) \cos \theta] v_i^L(\mathbf{x}) + [1 - (v_2/c_T) \cos \theta] v_i^T(\mathbf{x}) \} \quad (14)$$

where the v_i^α ($\alpha = L, T$) are the Cartesian components of the longitudinal and transverse particle velocities. They can be calculated by decomposing v_r and v_θ (equations (10) and (11)), multiplied by $[1 + (r/\bar{\rho})]^{-1/2}$, into their Cartesian components. The surface $\mathcal{S}_i$ is then taken as the wave front of the longitudinal or transverse part of (14). Lastly the surface integral is evaluated by the method of stationary phase. This is consistent with our previous far-field or high-frequency approximations.

Let us consider the special case of a circular rupture front of radius a . In this case the coordinate system (r, θ, ψ) shown in Figure 1 is a toroidal system which is related to the (x, y, z) system by the relations

$$x = -d \tan \phi + (a + r \cos \theta) \cos \psi \sin \phi + r \sin \theta \cos \phi - a \sin \phi \quad (15a)$$

$$y = d - (a + r \cos \theta) \cos \psi \cos \phi + r \sin \theta \sin \phi + a \cos \phi \quad (15b)$$

$$z = (a + r \cos \theta) \sin \psi \quad (15c)$$

Though the angle ϕ can lie between $\pi/2$ and $-\pi/2$, in practice we consider it to lie between $\pi/4$ and $-\pi/4$. The surface $\mathcal{S}_i$ can now be defined as

$$r = \bar{r} \quad -\psi_1 < \psi < \psi_2 \quad -\pi < \theta < \pi \quad (16)$$

We shall take the angles $\psi_1, \psi_2 < \pi$; only the upper part of the rupture front is considered significant to the excitation of Ray-

leigh waves. To further simplify the analysis we consider the point of observation to lie in the plane of reflection symmetry $z = 0$. Observation points not quite in the symmetry plane are considered in the closing part of Appendix B.

The stationary-phase points of (13b) for this particular case are

$$\psi = 0 \quad (17a)$$

$$\theta = \pi/2 - \phi - \theta_\alpha \quad \alpha = L, T \quad (17b)$$

where

$$\theta_\alpha = i \cosh^{-1}(c_\alpha/c_R) \quad \alpha = L, T \quad (18)$$

The most important details of the stationary-phase calculations are given in Appendix B. The end result of these calculations is a high-frequency approximation to the time-harmonic, Rayleigh wave displacement $u^R(x, y, z, \omega)$ that is given by the following expressions:

$$u^R(x, y, 0, \omega) = \mathbf{U}(\mathbf{x}) [g_L D^L(\pi/2 - \phi - \theta_L) \exp(-ik_L \bar{\mathbf{p}}^L \cdot \mathbf{x}_B) + \kappa^{-1} \eta g_T D^T(\pi/2 - \phi - \theta_T) \exp(-ik_T \bar{\mathbf{p}}^T \cdot \mathbf{x}_B)] \times D_0^{-1} e(\omega) (-i\omega)^{-1/2} (i) \quad (19)$$

where

$$\mathbf{U}(\mathbf{x}) = (2\kappa_R/\kappa^2) [d^L \exp(ik_L \mathbf{p}^L \cdot \mathbf{x}) + \kappa \eta d^T \exp(ik_T \mathbf{p}^T \cdot \mathbf{x})] \quad (20)$$

$$D^\alpha(\theta) = 4E_\alpha^{TV}(\theta) [1 - (v_2/c_\alpha) \cos \theta] \quad \alpha = L \quad (21a)$$

$$D^\alpha(\theta) = -4E_\alpha^{TV}(\theta) [1 - (v_2/c_\alpha) \cos \theta] \quad \alpha = T \quad (21b)$$

$$g_\alpha = \left[1 + \frac{d}{a} \left(\frac{x}{d} + \tan \phi \right) \frac{\sin(\phi + \theta_\alpha)}{\cos \theta_\alpha} \right]^{-1/2} \quad \alpha = L, T \quad (22)$$

$$\mathbf{p}^\alpha = \cos \theta_\alpha \mathbf{i} + \sin \theta_\alpha \mathbf{j} \quad \alpha = L, T \quad (23)$$

$$d^L = \mathbf{p}^L \quad (24a)$$

$$d^T = \mathbf{k} \times \mathbf{p}^T \quad (24b)$$

$$\mathbf{x} = x\mathbf{i} + y\mathbf{j} \quad (25a)$$

$$\mathbf{x}_B = -d \tan \phi \mathbf{i} + d\mathbf{j} \quad (25b)$$

The emission coefficients E_α^{TV} are given by (3a, 3b), $e(\omega)$ is given by (12) and D_0 is given by (A7). The overbars in (19) indicate the complex conjugate. The constant $\kappa = c_L/c_T$ and the constant $\eta = \cot 2\theta_T$. The terms g_α ($\alpha = L, T$), called the spreading factors, describe how the Rayleigh wave decays as it spreads across the surface of the half space. Because θ_α is complex, g_α is complex; accordingly, the branch of the square root in (22) is selected so that $\text{Re}(g_\alpha)$ is positive. Finally, note that we adopt the convention that the distance $x/d + \tan \phi$ always be positive; thus the observation point lies ahead of the rupture front when ϕ is positive and behind when ϕ is negative.

We are, however, primarily interested in the transient waveforms of the particle accelerations. Hence we first calculate the particle accelerations in the frequency domain, as described at the end of section 2, and then we invert the resulting expressions by using (9b). These operations give the following expressions for the Rayleigh wave particle acceleration $\mathbf{a}^R(x, y, z, t)$ in the symmetry plane:

$$\mathbf{a}^R(x, y, 0, t) = (2\kappa_R F_0/\kappa^2 D_0) (c_R/d)^{(4-\nu)/2} \cdot [a_1(x, y, t)\mathbf{i} + a_2(x, y, t)\mathbf{j}] \quad (26)$$

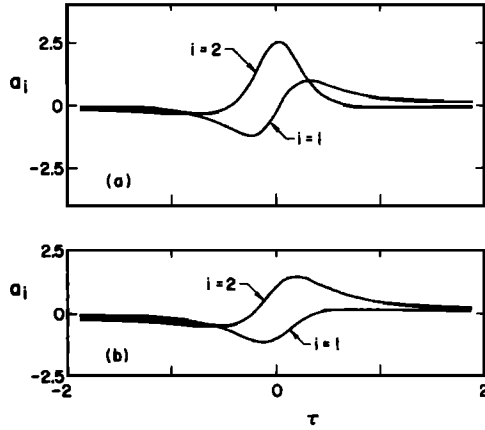


Fig. 2. The Rayleigh wave acceleration components a_i ($i = 1, 2$), evaluated at $\bar{y} = 0$ for a stopping event, are plotted against $\tau = \bar{t} - \bar{x}$. (a) The parameter $\nu = 1$ and (b) $\nu = 2$. For both 2a and 2b the angle $\phi = 0$ and $\bar{x} = 10$.

where

$$a_i(x, y, t) = \frac{4}{\pi} \Gamma\left(\frac{\nu+2}{2}\right) \Gamma\left(\frac{4-\nu}{2}\right) \sum_{\alpha, \beta=L, T} a_i^{\alpha\beta}(x, y, t) \quad (27)$$

$$a_i^{\alpha\beta}(x, y, t) = \frac{A_i^{\alpha\beta} \cos\{[(4-\nu)/2] \tan^{-1}(B^{\alpha\beta}) - \theta_i^{\alpha\beta} - (\nu\pi/4)\}}{(C^{\alpha\beta})^{(4-\nu)/2} [1 + (B^{\alpha\beta})^2]^{(4-\nu)/4}} \quad (28)$$

$$A_i^{\alpha\beta} \exp(i\theta_i^{\alpha\beta}) = \frac{S^{\alpha\beta} d_i^{\alpha} E_{\beta}^{TV} (\pi/2 - \phi - \theta_{\beta}) g_{\beta}}{[1 - (v_2/c_{\beta}) \sin(\phi + \theta_{\beta})]} \quad (29)$$

$$B^{\alpha\beta} = [(c_R t/d) - (x/d + \tan \phi)]/C^{\alpha\beta} \quad (30)$$

$$C^{\alpha\beta} = [1 - (c_R/c_{\alpha})^2]^{1/2}(y/d) + [1 - (c_R/c_{\beta})^2]^{1/2} \quad (31)$$

In equations (27)–(29) $i = 1, 2$, and in equations (28)–(31) $\alpha, \beta = L, T$. The functions E_{β}^{TV} are given by (3a, 3b) and the functions g_{β} are given by (22). The terms $S^{\alpha\beta}$ are given by

$$S^{LL} = +1 \quad (32a)$$

$$S^{TL} = +\kappa\eta \quad (32b)$$

$$S^{LT} = -\eta/\kappa \quad (32c)$$

$$S^{TT} = -\eta^2 \quad (32d)$$

the terms d_i^{α} are given by (24a, 24b), the term D_0 is given by (A7) and F_0 is a constant (section 2). The parameter ν must satisfy the inequalities $1 \leq \nu < 4$, where $\nu \geq 1$ for physical reasons (section 2), while $\nu < 4$ in order that the particular transform inversion used to calculate (26)–(28) remain valid. This range of ν encompasses all physically reasonable values.

4. DISCUSSION AND CONCLUSIONS

The emission coefficients E_{α}^{TV} ($\alpha = L, T$) (equations (3a, 3b)) show a double couple symmetry that is modified by the multiplicative factors F_{α} (equation (4)). Thus the strength of the emission depends directly upon the change in rupture front speed and inversely upon the terms $[1 - (v_1/c_{\alpha}) \cos \theta]$ and $[1 - (v_2/c_{\alpha}) \cos \theta]^{1/2}$. Inspection of F_{α} shows that $|E_T^{TV}| > |E_L^{TV}|$ for most values of θ . The emission coefficients, evaluated at $\theta = \pi/2 - \phi - \theta_{\beta}$ ($\beta = L, T$), affect the Rayleigh wave through the terms $A_i^{\alpha\beta}$ and $\theta_i^{\alpha\beta}$ ($i = 1, 2$) (equation (29)). Thus $|A_i^{TT}| > |A_i^{LL}|$, particularly near $\phi = 0$ where $|\cos 2\theta|$ is largest.

The arrival time, which follows from $B^{\alpha\beta} = 0$ (see equations

(28) and (30)), suggests that the Rayleigh wave emanates from a point directly above the rupture front. However, no Rayleigh wave can exist at the surface unless the incident waves properly couple to the surface [Ewing et al., 1957, pp. 63–66]. As a consequence a position at the surface where a Rayleigh wave is observed must satisfy one of the inequalities

$$(x/d) + \tan \phi > [(c_{\alpha}/c_R)^2 - 1]^{-1/2} \quad \alpha = L, T \quad (33)$$

Taking the more stringent of the two, and setting Poisson's ratio equal to 0.25, gives $(x/d) + \tan \phi > 2.3$. Because our calculation is a far-field one which is accurate only near times close to the arrival time, this inequality is always satisfied. However, it does set a lower limit. For Poisson's ratio = 0.25 and $\phi = -15^\circ$, we find $x/d > 2.6$. In an earlier paper Harris and Achenbach [1981] estimated that body wave reflections at the surface are important only when $x/d < 4$, for the same values of ϕ and Poisson's ratio.

The depth dependence of the Rayleigh wave manifests itself through the normalizing term $(c_R/d)^{(4-\nu)/2}$ (equation (26)) and through the term $(C^{\alpha\beta})^{(\nu-4)/2}$ (equations (28) and (31)). Note that for $\nu = 1$, the decay with depth is greatest. But the depth dependence also manifests itself in another way: d is the natural normalizing distance. The normalized time is $\bar{t} = (c_R t)/d$, and the normalized lengths are $\bar{x} = x/d$ and $\bar{y} = y/d$. This leaves only the ratio $\bar{d} = d/a$ in g_{β} ($\beta = L, T$) (equation (22)).

In the plots of the results two conventions regarding ϕ need to be noted: (1) The angle ϕ can take positive and negative values but $x/d + \tan \phi$ remains positive. This, however, means that θ is positive in a downward sense for ϕ positive, but is positive in an upward sense for ϕ negative. (2) The horizontal distance from the rupture front in the plane of symmetry is $x + d \tan \phi$ so that when ϕ is varied this distance changes even though x is fixed. In all the plots the following parameters are assigned fixed values: $v_1/c_T = 0.5$, $v_2/c_T = 0$, $\bar{y} = 0$, $\bar{d} = 0.5$, and Poisson's ratio = 0.25.

Figure 2 shows the components of acceleration a_i ($i = 1, 2$) for a stopping event plotted against $\tau = \bar{t} - \bar{x}$ for (Figure 2a) $\nu = 1$ and (Figure 2b) $\nu = 2$. The angle $\phi = 0^\circ$ and $\bar{x} = 10$. The values $\nu = 1$ and $\nu = 2$ correspond to brittle fracture and to some yielding at the rupture front respectively. Comparison of Figures 2a with 2b shows that increasing ν reduces the sharpness of the signal though the basic waveform remains unchanged.

Figure 3 shows the components of acceleration a_i ($i = 1, 2$) for a stopping event plotted against $\tau = \bar{t} - \bar{x}$ for (Figure 3a) $\phi = 30^\circ$, $\bar{x} = 5$; (Figure 3b) $\phi = 0^\circ$, $\bar{x} = 5$; (Figure 3c) $\phi = -30^\circ$, $\bar{x} = 5$; and (Figure 3d) $\phi = -30^\circ$, $\bar{x} = 20$. The parameter $\nu = 1$ throughout. Comparison of Figures 3a, 3b, and 3c shows that $\phi = 0$ and $\phi = -30^\circ$ both produce stronger signals than $\phi = 30^\circ$. This is caused, in part, by the normalized distance $\bar{x} + \tan \phi$ getting smaller. In addition, however, the $\phi = 0$ case is strong because, as discussed above, $|E_T^{TV}|$ is large near $\pi/2 - \theta_T$ (recall θ_T is complex), and the $\phi = -30^\circ$ case is strong because, as shall be shown below, $|g_T|$ is large near $\bar{x} = 5$. Comparison of either Figure 2a with 3b or Figure 3c with 3d shows the geometrical decay of the signals as $\bar{x}$ becomes larger.

Figure 4 shows the magnitude of the spreading factors g_{β} ($\beta = L, T$) (equation (22)) plotted against $\bar{x}$ for $\phi = -30^\circ$. Note that both $|g_L|$ and $|g_T|$ go through a maximum and then start to fall off approximately as $(\bar{x})^{-1/2}$ (the geometrical decay for a Rayleigh wave excited by a buried point source). These maxima are the result of a focusing effect caused by the curved rupture front. Note that the region in which Rayleigh waves can exist

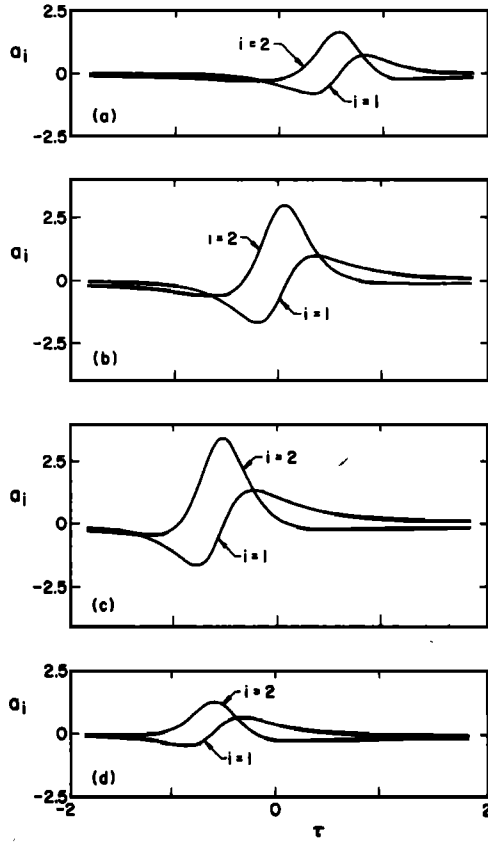


Fig. 3. The Rayleigh wave acceleration components a_i ($i = 1, 2$), evaluated at $\bar{y} = 0$ for a stopping event, are plotted against $\tau = \bar{t} - \bar{x}$. The angle ϕ and distance $\bar{x}$ take the following values: (a) $\phi = 30^\circ$, $\bar{x} = 5$; (b) $\phi = 0$, $\bar{x} = 5$; (c) $\phi = -30^\circ$, $\bar{x} = 5$; (d) $\phi = -30^\circ$, $\bar{x} = 20$. The parameter $\nu = 1$ throughout.

(see above) is $\bar{x} > 2.9$ and in that region only $|g_T|$ has a maximum.

In Figures 2 and 3 we considered a single stopping event. In Figure 5 we have combined a starting and stopping event. The Rayleigh wave particle acceleration for this combined event is still given by (26). However, (27) needs to be rewritten as

$$a_i(x, y, t) = \frac{4}{\pi} \Gamma\left(\frac{\nu+2}{2}\right) \Gamma\left(\frac{4-\nu}{2}\right) \sum_{\alpha, \beta=L, T} \left[-\left(\frac{d}{d_1}\right)^{(4-\nu)/2} a_{i1}^{\alpha\beta}(x, y, t) \right.$$

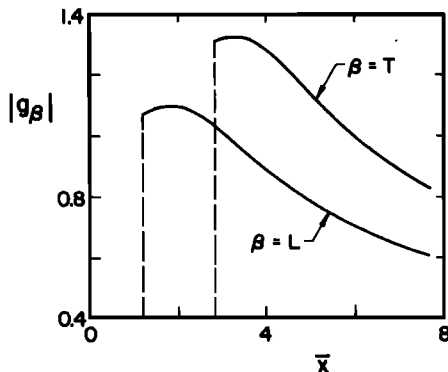


Fig. 4. The magnitude of the spreading factor $|g_\beta|$ ($\beta = L, T$) is plotted against $\bar{x}$, where $\bar{x}$ satisfies (33). The angle $\phi = -30^\circ$.

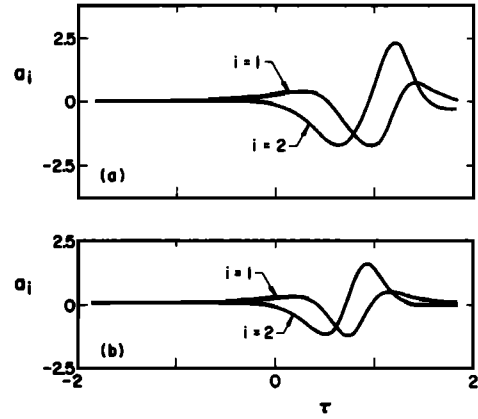


Fig. 5. The Rayleigh wave acceleration components a_i ($i = 1, 2$), evaluated at $\bar{y} = 0$, are plotted against $\tau = \bar{t} - \bar{x}$ for a combined starting and stopping event. (a) The parameter $\delta = 0.1$ and (b) $\delta = 0.05$. The angle $\phi = 30^\circ$, $\bar{x} = 5$, and $\nu = 1$ for both.

$$+ \left(\frac{d}{d_2}\right)^{(4-\nu)/2} a_{i2}^{\alpha\beta}(x, y, t - 2\delta/\nu) \Big] \quad i = 1, 2 \quad (34)$$

The components $a_{i1}^{\alpha\beta}$ are given by (28)–(31) with d replaced by $d_1 = d + \delta \cos \phi$, and the components $a_{i2}^{\alpha\beta}$ are given by (28)–(31) with d replaced by $d_2 = d - \delta \cos \phi$ and t replaced by $t - (2\delta/\nu)$. The distance 2δ is the distance the rupture front travels at a constant speed v after starting at $t = 0$. The normalized δ is $\delta = \delta/a$. The distance d is the depth of the midpoint and is used as the normalizing distance. Figure 5 shows the acceleration components a_i ($i = 1, 2$) plotted against $\tau = \bar{t} - \bar{x}$ for (Figure 5a) $\delta = 0.1$ and (Figure 5b) $\delta = 0.05$. The angle $\phi = 30^\circ$ and $\bar{x} = 5$. Comparison of Figure 3a with Figures 5a, 5b shows that the signals of the combined event are essentially the derivatives of the single event.

The results of this paper lead to the following conclusions:

1. An analytical method to approximate the Rayleigh waves excited by starting and stopping phases of a fault plane rupture process has been developed.
2. From a mathematical viewpoint, the L and T rays emitted by the rupture process that excite the Rayleigh wave must leave the fault plane under the complex angles $\theta = \pi/2 - \phi - \theta_L$ and $\theta = \pi/2 - \phi - \theta_T$. Thus the emitted waves contribute to the phase of the Rayleigh wave through the terms $\theta_i^{\alpha\beta}$ ($i = 1, 2$; $\alpha, \beta = L, T$) as well as to the amplitude.
3. The transition from a region on the surface where body waves dominate to that where Rayleigh waves dominate occurs for values of x/d near 3 or 4 (for small values of ϕ and Poisson's ratio = 0.25).
4. An increase in the parameter ν , which means increasing yielding at the rupture front, decreases the maximum particle acceleration of the Rayleigh wave at the surface.
5. The strongest Rayleigh waves occur when the plane of rupture is nearly perpendicular to the surface or when the point of observation lies near a focal region. Thus the curvature of the rupture front can influence the maximum particle acceleration at the surface.

APPENDIX A: THE RAYLEIGH-WAVE COMPONENT OF THE GREEN'S TENSOR

Consider an elastic half space with a traction-free boundary. The Green's tensor $u_{i,k}^G$ is the displacement excited there by a

time-harmonic point load applied at $\mathbf{x} = \mathbf{x}'$ in the direction $\mathbf{a}_k$. The tensor $\tau_{ij;k}^{GR}$ is the associated stress. Both tensors are derived in Achenbach *et al.*, [1982, chapter 3]. The Rayleigh wave contribution to $u_{i;k}^{GR}$ is given by

$$u_{i;k}^{GR}(\mathbf{x}, \mathbf{x}') = A_0 [k_R \mathbf{p} \cdot (\mathbf{x} - \mathbf{x}')]^{-1/2} U_L(\mathbf{x}) \bar{U}_k(\mathbf{x}') \quad (\text{A1})$$

where

$$U(\mathbf{x}) = (2\kappa_R/\kappa^2) [\mathbf{d}^L \exp(ik_L \mathbf{p}^L \cdot \mathbf{x}) + \kappa\eta \mathbf{d}^T \exp(ik_T \mathbf{p}^T \cdot \mathbf{x})] \quad (\text{A2})$$

The overbar indicates the complex conjugate. This is an asymptotic result which is only accurate if $|k_R \mathbf{p} \cdot (\mathbf{x} - \mathbf{x}')| \gg 1$. The various ancillary formulae are

$$A_0 = (e^{i\pi/4} k_L) / [\mu(2\pi)^{1/2} D_0] \quad (\text{A3})$$

$$\mathbf{p} = [(x - x')\mathbf{i} + (z - z')\mathbf{k}] / [(x - x')^2 + (z - z')^2]^{1/2} \quad (\text{A4})$$

$$\mathbf{p}^\alpha = \cos \theta_\alpha \mathbf{p} + \sin \theta_\alpha \mathbf{j} \quad \alpha = L, T \quad (\text{A5})$$

$$\mathbf{d}^L = \mathbf{p}^L \quad (\text{A6a})$$

$$\mathbf{d}^T = (\mathbf{p} \times \mathbf{j}) \times \mathbf{p}^T \quad (\text{A6b})$$

$$D_0 = [\kappa_R \kappa^2 (\kappa_R^2 - \kappa^2)^{1/2}]^{-1} \frac{d}{dt} [4t^2(t^2 - \kappa^2)^{1/2} (t^2 - 1)^{1/2} - (\kappa^2 - 2t^2)^2] \quad (\text{A7})$$

where $t = \kappa_R$ in (A7), and

$$\theta_\alpha = i \cosh^{-1}(c_\alpha/c_R) \quad \alpha = L, T \quad (\text{A8a})$$

$$\eta = \cot 2\theta_T \quad (\text{A8b})$$

Also, $\kappa = c_L/c_T$ and $\kappa_R = c_L/c_R$. Wave numbers are defined by $k_\alpha = \omega/c_\alpha$ ($\alpha = L, T, R$) where ω is the angular frequency and c_α ($\alpha = L, T, R$) are the L , T , and Rayleigh wave speeds.

APPENDIX B: EVALUATION OF THE REPRESENTATION INTEGRAL

The surface of integration $\mathcal{S}_i$ of (13b) is the toroidal surface (16) (Figure 1). Using the Rayleigh wave Green's tensors $u_{i;k}^{GR}$ and $\tau_{ij;k}^{GR}$, where the former is given by (A1), in combination with the displacement u_i^I given by (14), we write the representation integral (13b) as a sum of eight integrals each of which has in its integrand the term $\exp[i(k_\alpha \mathbf{p}^\alpha \cdot \mathbf{x}' - k_\beta \bar{\mathbf{p}}^\beta \cdot \mathbf{x})]$ ($\alpha, \beta = L, T$), where the overbar indicates the complex conjugate. The vectors $\mathbf{p}^\alpha$ are given by (A5) with the coordinates (x, z) and (x', z') interchanged. It can be shown that the physically relevant stationary-phase point of each integrand occurs at that point $\mathbf{x}'$ for which $\mathbf{n} = \mathbf{p}^\alpha$ ($\alpha = L, T$), where $-\mathbf{n}$ is a unit vector in the r direction (Figure 1), and that as a consequence only four integrals are nonzero at the stationary-phase point. Each of these integrals has the basic form

$$I^{\alpha\beta} = \frac{(k_R k_\alpha)^{1/2}}{2\pi i} \int_{-\pi}^{\pi} \int_{\psi_1}^{\psi_2} \left[\frac{(a\bar{r})(a + \bar{r} \cos \theta)}{f(\theta, \psi)} \right]^{1/2} \cdot H^\alpha(\mathbf{x}', \mathbf{x}) D^\alpha(\theta) \exp\{ik_\alpha[\bar{r} + F^{\alpha\beta}(\theta, \psi)]\} d\psi d\theta \quad \alpha, \beta = L, T \quad (\text{B1})$$

The functions H^α ($\alpha = L, T$) assume the simple forms

$$H^L = 1 \quad (\text{B2a})$$

$$H^T = \eta \quad (\text{B2b})$$

at the stationary-phase points $\mathbf{n} = \mathbf{p}^L$ and $\mathbf{n} = \mathbf{p}^T$, respectively.

The functions D^α ($\alpha = L, T$) are given by (21a, 21b). The toroidal coordinates (r, θ, ψ) are related to the Cartesian coordinates (x, y, z) by (15a, 15b, 15c). However, it is necessary to introduce yet another coordinate system, namely $x = \chi \cos \varepsilon$ and $z = \chi \sin \varepsilon$ with y unchanged. With these new coordinates we can express $f(\theta, \psi)$ and $F^{\alpha\beta}(\theta, \psi)$ as follows:

$$f(\theta, \psi) = \{[(a + \bar{r} \cos \theta) \sin \psi - \chi \sin \varepsilon]^2 + [(a + \bar{r} \cos \theta) \cos \psi \sin \phi + \bar{r} \sin \theta \cos \phi - a \sin \phi - d \tan \phi - \chi \cos \varepsilon]^2\}^{1/2} \quad (\text{B3})$$

$$F^{\alpha\beta}(\theta, \psi) = f(\theta, \psi) \cos \theta_\alpha + [\bar{r} \sin \theta \sin \phi - (a + \bar{r} \cos \theta) \cos \psi \cos \phi] \sin \theta_\alpha + (d + a \cos \phi) \sin \theta_\alpha + y \cos \theta_\alpha \tan \theta_\beta \quad \alpha, \beta = L, T \quad (\text{B4})$$

The stationary-phase points of the integral (B1) are found by solving the two simultaneous equations $F_\theta = 0$ and $F_\psi = 0$ for θ and ψ . In general, these are complicated transcendental equations that can only be solved explicitly when $\varepsilon = 0$ (i.e., when the observation point lies in the plane of symmetry). For $\varepsilon = 0$, the stationary-phase point of interest is given by $\psi = 0$ and $\theta = \pi/2 - \phi - \theta_\alpha$ ($\alpha = L, T$). Performing the stationary-phase calculation we obtain

$$I^{\alpha\beta} = g_\alpha D^\alpha(\pi/2 - \phi - \theta_\alpha) H^\alpha \cdot \exp[i(k_\alpha \mathbf{p}^\alpha \cdot \mathbf{x}_B - k_\beta \bar{\mathbf{p}}^\beta \cdot \mathbf{x})] \quad \alpha, \beta = L, T \quad (\text{B5})$$

where g_α , D^α , and H^α are given by (22), (21a), (21b), and (B2a), (B2b), respectively. The vectors $\mathbf{x}$ and $\mathbf{x}_B$ are given by (25a), (25b). Note that $\mathbf{p} = -\mathbf{i}$ in this appendix whereas $\mathbf{p} = \mathbf{i}$ in (23).

The complex factors g_α in (B5) express the focusing effect that the slip-zone geometry has upon the Rayleigh wave. It can be shown that

$$-[(\pi/2) + \tan^{-1}(\tan \phi \cot \theta_\alpha)]/2 < \arg(g_\alpha) < 0 \quad (\text{B6})$$

$\alpha = L, T$

The upper limit occurs as $a \rightarrow \infty$, while the lower limit occurs as $x \rightarrow \infty$.

Let us now assume that the observation point $\mathbf{x}$ has a small z component. Thus the angle ε is small and the equations $F_\psi = 0$, $F_\theta = 0$ can be solved for values of ψ and θ , correct to $O(\varepsilon^2)$. The critical value of θ remains unchanged, while $\psi = \varepsilon g_\alpha^2(x/a)$ ($\alpha = L, T$). The phase of (B5) is thus changed. However, its amplitude remains unchanged to $O(\varepsilon^2)$. Note that ψ is complex because the g_α are complex; that is, the 'flash point' moves into complex space.

APPENDIX C: THE FUNCTION $S_-(\zeta)$

The function $S_-(\zeta)$ is given by

$$\ln S_-(\zeta) = -\frac{1}{\pi} \int_a^b \tan^{-1} \left[\frac{4z^2 |\gamma_L| |\gamma_T|}{(2z^2 - s_T^2 - s_T^2 z^2/s_2^2 + 2s_T^2 z/s_2)^2} \right] \frac{dz}{z - \zeta} \quad (\text{C1})$$

where

$$\gamma_\alpha = [s_\alpha^2(1 - z/s_2)^2 - z^2]^{1/2} \quad \alpha = L, T \quad (\text{C2})$$

$$a = s_L/(1 + s_L/s_2) \quad (\text{C3a})$$

$$b = s_T/(1 + s_T/s_2) \quad (\text{C3b})$$

Here $s_L = 1/c_L$, $s_T = 1/c_T$, $s_1 = 1/v_1$, and $s_2 = 1/v_2$. When $a < \zeta < b$, ζ should approach the real line through positive imaginary values.

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